velocity into the radius equation to equate the pitch and radius:

$$\begin{aligned} p &= r \\ v_{\parallel}\, T &= \frac{mv_{\perp}}{qB} \\ v\cos\theta\frac{2\pi m}{qB} &= \frac{mv\sin\theta}{qB} \\ 2\pi &= \tan\theta \\ \theta &= 81.0°. \end{aligned}$$

Significance

If this angle were $0°$, only parallel velocity would occur and the helix would not form, because there would be no circular motion in the perpendicular plane. If this angle were $90°$, only circular motion would occur and there would be no movement of the circles perpendicular to the motion. That is what creates the helical motion.

11.4 | Magnetic Force on a Current-Carrying Conductor

Learning Objectives

By the end of this section, you will be able to:

- Determine the direction in which a current-carrying wire experiences a force in an external magnetic field
- Calculate the force on a current-carrying wire in an external magnetic field

Moving charges experience a force in a magnetic field. If these moving charges are in a wire—that is, if the wire is carrying a current—the wire should also experience a force. However, before we discuss the force exerted on a current by a magnetic field, we first examine the magnetic field generated by an electric current. We are studying two separate effects here that interact closely: A current-carrying wire generates a magnetic field and the magnetic field exerts a force on the current-carrying wire.

Magnetic Fields Produced by Electrical Currents

When discussing historical discoveries in magnetism, we mentioned Oersted's finding that a wire carrying an electrical current caused a nearby compass to deflect. A connection was established that electrical currents produce magnetic fields. (This connection between electricity and magnetism is discussed in more detail in **Sources of Magnetic Fields**.)

The compass needle near the wire experiences a force that aligns the needle tangent to a circle around the wire. Therefore, a current-carrying wire produces circular loops of magnetic field. To determine the direction of the magnetic field generated from a wire, we use a second right-hand rule. In RHR-2, your thumb points in the direction of the current while your fingers wrap around the wire, pointing in the direction of the magnetic field produced (**Figure 11.11**). If the magnetic field were coming at you or out of the page, we represent this with a dot. If the magnetic field were going into the page, we represent this with an $\times$. These symbols come from considering a vector arrow: An arrow pointed toward you, from your perspective, would look like a dot or the tip of an arrow. An arrow pointed away from you, from your perspective, would look like a cross or an $\times$. A composite sketch of the magnetic circles is shown in **Figure 11.11**, where the field strength is shown to decrease as you get farther from the wire by loops that are farther separated.

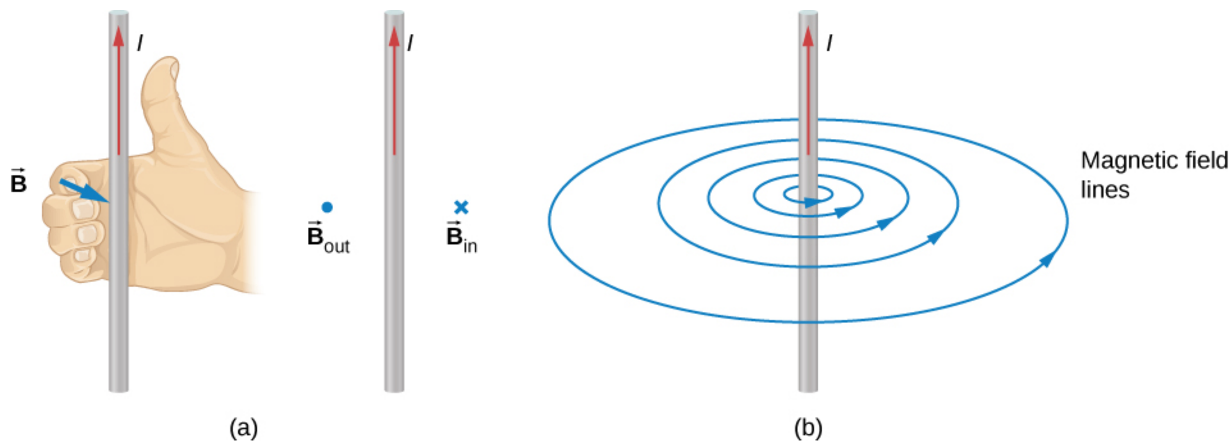

Figure 11.11 (a) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow). (b) A long and straight wire creates a field with magnetic field lines forming circular loops.

Calculating the Magnetic Force

Electric current is an ordered movement of charge. A current-carrying wire in a magnetic field must therefore experience a force due to the field. To investigate this force, let's consider the infinitesimal section of wire as shown in **Figure 11.12**. The length and cross-sectional area of the section are dl and A, respectively, so its volume is $V = A \cdot dl$. The wire is formed from material that contains n charge carriers per unit volume, so the number of charge carriers in the section is $nA \cdot dl$. If the charge carriers move with drift velocity $\vec{\mathbf{v}}_{\mathrm{d}}$, the current I in the wire is (from **Current and Resistance**)

$$I = neAv_d.$$

The magnetic force on any single charge carrier is $e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}$, so the total magnetic force $d\vec{\mathbf{F}}$ on the $nA \cdot dl$ charge carriers in the section of wire is

$$d\vec{\mathbf{F}} = (nA \cdot dl)e\vec{\mathbf{v}}_{\mathrm{d}} \times \vec{\mathbf{B}}. \tag{11.10}$$

We can define dl to be a vector of length dl pointing along $\vec{\mathbf{v}}_{\mathrm{d}}$, which allows us to rewrite this equation as

$$d\vec{\mathbf{F}} = neAv_{\mathrm{d}}\vec{\mathbf{dl}} \times \vec{\mathbf{B}}, \tag{11.11}$$

or

$$d\vec{\mathbf{F}} = I\vec{\mathbf{dl}} \times \vec{\mathbf{B}}. \tag{11.12}$$

This is the magnetic force on the section of wire. Note that it is actually the net force exerted by the field on the charge carriers themselves. The direction of this force is given by RHR-1, where you point your fingers in the direction of the current and curl them toward the field. Your thumb then points in the direction of the force.

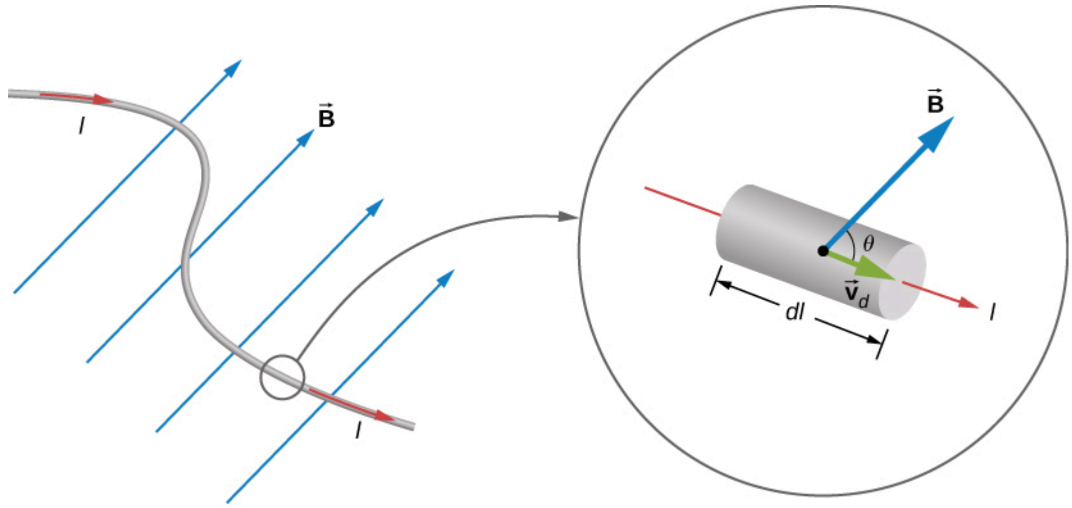

Figure 11.12 An infinitesimal section of current-carrying wire in a magnetic field.

To determine the magnetic force $\vec{\mathbf{F}}$ on a wire of arbitrary length and shape, we must integrate Equation 11.12 over the entire wire. If the wire section happens to be straight and B is uniform, the equation differentials become absolute quantities, giving us

$$\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}. \tag{11.13}$$

This is the force on a straight, current-carrying wire in a uniform magnetic field.

Example 11.4

Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire

A wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 11.13). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

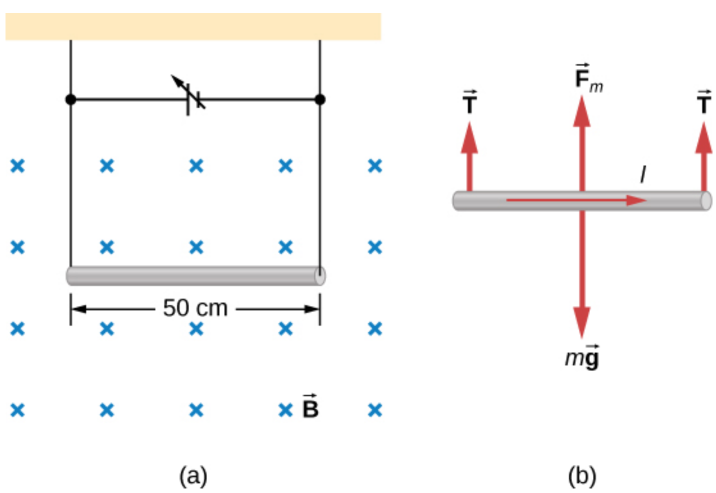

Figure 11.13 (a) A wire suspended in a magnetic field. (b) The free-body diagram for the wire.

Strategy

From the free-body diagram in the figure, the tensions in the supporting leads go to zero when the gravitational and magnetic forces balance each other. Using the RHR-1, we find that the magnetic force points up. We can then determine the current I by equating the two forces.

Solution

Equate the two forces of weight and magnetic force on the wire:

$$mg = IlB.$$

Thus,

$$I = \frac{mg}{lB} = \frac{(0.010\ \text{kg})(9.8\ \text{m/s}^2)}{(0.50\ \text{m})(0.50\ \text{T})} = 0.39\ \text{A}.$$

Significance

This large magnetic field creates a significant force on a length of wire to counteract the weight of the wire.

Example 11.5

Calculating Magnetic Force on a Current-Carrying Wire

A long, rigid wire lying along the y-axis carries a 5.0-A current flowing in the positive y-direction. (a) If a constant magnetic field of magnitude 0.30 T is directed along the positive x-axis, what is the magnetic force per unit length on the wire? (b) If a constant magnetic field of 0.30 T is directed 30 degrees from the $+x$-axis towards the $+y$-axis, what is the magnetic force per unit length on the wire?

Strategy

The magnetic force on a current-carrying wire in a magnetic field is given by $\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$. For part a, since the current and magnetic field are perpendicular in this problem, we can simplify the formula to give us the magnitude and find the direction through the RHR-1. The angle θ is 90 degrees, which means $\sin\theta = 1$. Also, the length can be divided over to the left-hand side to find the force per unit length. For part b, the current times length is written in unit vector notation, as well as the magnetic field. After the cross product is taken, the directionality is evident by the resulting unit vector.

Solution

a. We start with the general formula for the magnetic force on a wire. We are looking for the force per unit length, so we divide by the length to bring it to the left-hand side. We also set $\sin\theta = 1$. The solution therefore is

$$\begin{aligned} F &= IlB\sin\theta \\ \frac{F}{l} &= (5.0\ \text{A})(0.30\ \text{T}) \\ \frac{F}{l} &= 1.5\ \text{N/m}. \end{aligned}$$

Directionality: Point your fingers in the positive y-direction and curl your fingers in the positive x-direction. Your thumb will point in the $-\vec{\mathbf{k}}$ direction. Therefore, with directionality, the solution is

$$\frac{\vec{\mathbf{F}}}{l} = -1.5\,\vec{\mathbf{k}}\ \text{N/m}.$$

b. The current times length and the magnetic field are written in unit vector notation. Then, we take the cross product to find the force:

$$\begin{aligned} \vec{\mathbf{F}} &= I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}} = (5.0A)l\,\hat{\mathbf{j}} \times \left(0.30T\cos(30°)\,\hat{\mathbf{i}} + 0.30T\sin(30°)\,\hat{\mathbf{j}}\right) \\ \vec{\mathbf{F}}/l &= -1.30\,\hat{\mathbf{k}}\text{ N/m.} \end{aligned}$$

Significance

This large magnetic field creates a significant force on a small length of wire. As the angle of the magnetic field becomes more closely aligned to the current in the wire, there is less of a force on it, as seen from comparing parts a and b.

11.3 Check Your Understanding A straight, flexible length of copper wire is immersed in a magnetic field that is directed into the page. (a) If the wire's current runs in the +*x*-direction, which way will the wire bend? (b) Which way will the wire bend if the current runs in the –*x*-direction?

Example 11.6

Force on a Circular Wire

A circular current loop of radius R carrying a current I is placed in the *xy*-plane. A constant uniform magnetic field cuts through the loop parallel to the *y*-axis (**Figure 11.14**). Find the magnetic force on the upper half of the loop, the lower half of the loop, and the total force on the loop.

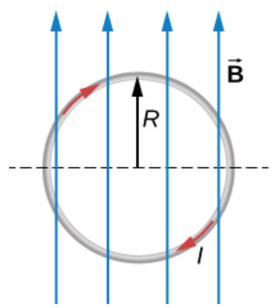

Figure 11.14 A loop of wire carrying a current in a magnetic field.

Strategy

The magnetic force on the upper loop should be written in terms of the differential force acting on each segment of the loop. If we integrate over each differential piece, we solve for the overall force on that section of the loop. The force on the lower loop is found in a similar manner, and the total force is the addition of these two forces.

Solution

A differential force on an arbitrary piece of wire located on the upper ring is:

$$dF = IB\sin\theta\, dl.$$

where θ is the angle between the magnetic field direction (+*y*) and the segment of wire. A differential segment is located at the same radius, so using an arc-length formula, we have:

$$\begin{aligned} dl &= R\,d\theta \\ dF &= IBR\sin\theta\, d\theta. \end{aligned}$$

In order to find the force on a segment, we integrate over the upper half of the circle, from 0 to π. This results in:

$$F = IBR\int_0^{\pi}\sin\theta\, d\theta = IBR(-\cos\pi + \cos 0) = 2IBR.$$

The lower half of the loop is integrated from π to zero, giving us:

$$F = IBR\int_{\pi}^{0} \sin\theta d\theta = IBR(-\cos 0 + \cos\pi) = -2IBR.$$

The net force is the sum of these forces, which is zero.

Significance

The total force on any closed loop in a uniform magnetic field is zero. Even though each piece of the loop has a force acting on it, the net force on the system is zero. (Note that there is a net torque on the loop, which we consider in the next section.)

11.5 | Force and Torque on a Current Loop

Learning Objectives

By the end of this section, you will be able to:

- Evaluate the net force on a current loop in an external magnetic field
- Evaluate the net torque on a current loop in an external magnetic field
- Define the magnetic dipole moment of a current loop

Motors are the most common application of magnetic force on current-carrying wires. Motors contain loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted into mechanical work in the process. Once the loop's surface area is aligned with the magnetic field, the direction of current is reversed, so there is a continual torque on the loop (**Figure 11.15**). This reversal of the current is done with commutators and brushes. The commutator is set to reverse the current flow at set points to keep continual motion in the motor. A basic commutator has three contact areas to avoid and dead spots where the loop would have zero instantaneous torque at that point. The brushes press against the commutator, creating electrical contact between parts of the commutator during the spinning motion.

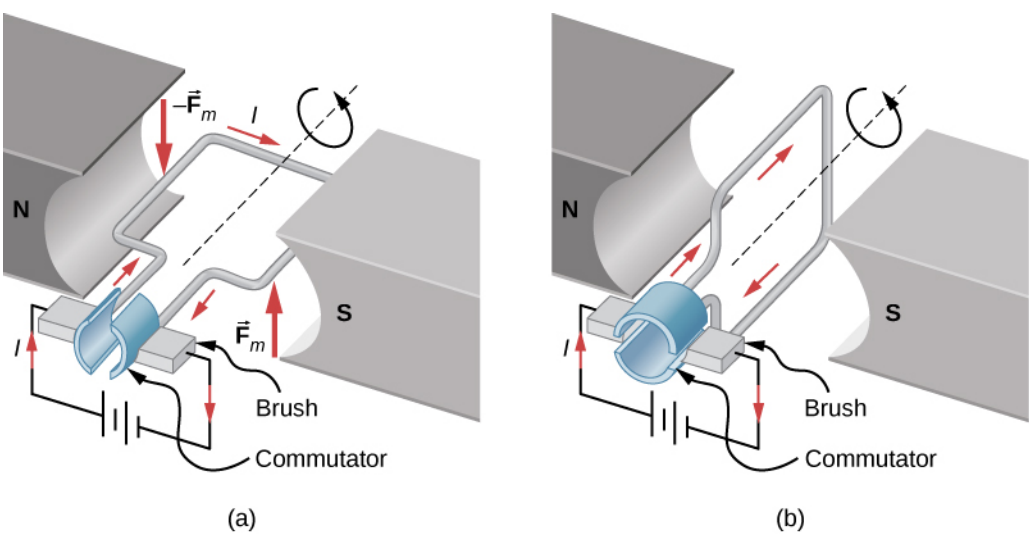

Figure 11.15 A simplified version of a dc electric motor. (a) The rectangular wire loop is placed in a magnetic field. The forces on the wires closest to the magnetic poles (N and S) are opposite in direction as determined by the right-hand rule-1. Therefore, the loop has a net torque and rotates to the position shown in (b). (b) The brushes now touch the commutator segments so that no current flows through the loop. No torque acts on the loop, but the loop continues to spin from the initial velocity given to it in part (a). By the time the loop flips over, current flows through the wires again but now in the opposite direction, and the process repeats as in part (a). This causes continual rotation of the loop.